THE IMPACT OF ADVERTISING ON MEDIA BIAS

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Abstract

In this study, the authors investigate the role of advertising in affecting the extent of bias in the media. When making advertising choices, advertisers evaluate both the size and the composition of the readership of the different outlets. The profile of the readers matters since advertisers wish to target readers who are likely to be receptive to their advertising messages. It is demonstrated that when advertising supplements subscription fees, it may serve as a polarizing or moderating force, contingent upon the extent of heterogeneity among advertisers. When heterogeneity is large, each advertiser chooses a single outlet for placing ads (Single-Homing), and greater polarization arises in comparison to the case that media relies on subscription fees only for revenues. In contrast, when heterogeneity is small, each advertiser chooses to place ads in multiple outlets (Multi-Homing), and reduced polarization results. For intermediate levels of heterogeneity, some advertisers choose to Single-Home and others choose to Multi-Home.

Keywords: Media Competition; Bias in News; Advertising; Two-Sided Markets

1. INTRODUCTION

Bias in news media is well known (e.g., Groseclose and Milyo 2005, and Hamilton 2004) and can be defined as selective omission, choice of words and varying credibility ascribed to the primary source (Gentzkow and Shapiro 2006). Reasons for the existence of media bias range from journalists' desire to enhance their career opportunities (Baron 2006) to media's incentive to increase audience ratings (Bernhardt, Krasa and Polborn 2008). In a recent paper by Mullainathan and Shleifer (MS 2005), a link is established between subscription fees and media bias. By assuming that readers prefer news consistent with their beliefs and that newspapers can slant toward these beliefs, MS (2005) show that when the papers' sole source of revenue is from subscription fees (i.e., price for news), they slant news toward extreme positions.

For many media outlets, however, 60% to 80% of total revenue stems from advertising (Strömberg 2004), as opposed to subscription. Thus, in this study, we aim to complement the work of MS (2005) by recognizing that newspapers rely on revenues that accrue both from subscription fees paid by readers and advertising fees paid by advertisers. We investigate how the existence of these two sources of revenue affect the extent of bias in reporting that is selected by the media. Similar to MS (2005), we assume that readers enjoy news confirming their beliefs, and newspapers slant toward these beliefs. This assumption is consistent with Iyengar and Hahn's (2009) recent evidence of ideological selectivity in media use. These authors show that while conservatives prefer to follow Fox News and to avoid news from CNN and NPR, liberals follow CNN and NPR, but avoid Fox News.

In order to understand the role of advertising in determining the extent of competition between newspapers, we specify in the model the effectiveness of advertisements to enhance product perceptions. We argue that this effectiveness, for some products, may depend upon the political beliefs of readers of the ads. It has been long established in the Consumer Behavior literature that products reflect a person's self-concept (Belk 1988). They provide a way for a person to express her self-image, which may be strongly correlated with her political beliefs. We introduce, therefore, a product specific variable that measures the extent to which political beliefs play a role in enhancing consumer perceptions of the product when it is advertised. While for some products this measure is significant, for others it is trivial. For example, while "green" products, such as Toyota Prius, or Apple's Mac computer[1] may appeal more to liberals, "American" products, such as the Chevy Truck, may appeal more to conservative consumers. However, there are many other products, such as automobile tires or insurance products, for which political beliefs and product perceptions may not be related to a large extent[2]. Therefore, ads for these products may not have differential effects on consumers with different political beliefs.

The variable that we introduce to measure the correlation between the beliefs of readers and the effectiveness of advertising in enhancing perceptions is distributed in our model over a bounded interval. The length of this interval captures the extent of heterogeneity among advertisers, with longer intervals indicating significant differences in the appeal of products to liberal vs. conservative readers. In our model we show that this degree of heterogeneity among advertisers plays a role in determining whether advertisers choose to place ads with a single newspaper or with both newspapers. The literature on two-sided markets has referred to these two possible outcomes as Single and Double-Homing by advertisers, respectively (See Armstrong (2006), for instance.) While Single-Homing arises as the unique equilibrium when the extent of heterogeneity is large, Double-Homing arises when it is small. For intermediate levels of heterogeneity, some advertisers Single-Home and others Double-Home.

We further investigate the manner in which the advertisers' choice between the newspapers affects the slanting strategies of media outlets. In particular, we ask whether in their attempt to attract advertisers, media choose to polarize or mitigate their slanting in reporting the news. We show that when advertising is their only source of revenue, newspapers choose to eliminate any slanting in their reporting in order to appeal to readers of moderate beliefs. This result is in sharp contrast to the extreme bias reported in MS (2005). When newspapers rely both on advertising and subscription fees, advertising can serve as a polarizing or moderating force in affecting the reporting of newspapers through three effects. First, adding the advertising market puts downward pressure on subscription fees, as newspapers intensify their competition for subscribers in order to attract advertisers. Hence, newspapers may have a stronger incentive to polarize in order to further differentiate their positions and alleviate such competition. Second, adding the advertising market implies that newspapers reduce their reliance on subscribers in favor of advertisers. As a result, they may choose less slanting in their reporting strategies to improve their appeal to moderate readers, and by doing so, offer a bigger readership to advertisers. Finally, advertisers wish to target readers who are more receptive to their advertising messages, thus providing stronger incentives for newspapers to polarize to establish greater distinctiveness and offer a better match between readers and advertisers.

We demonstrate that at the equilibrium with Double-Homing the size of the readership is the dominating factor influencing advertisers, leading to reduced polarization in news reporting when advertising is added as a source of revenue to supplement subscription fees. In contrast, at the equilibrium with Single-Homing, the objective of targeting the "right" consumers reinforces the objective of alleviating downward pressures on subscription fees, to yield greater polarization in reporting when advertising supplements subscription fees.

There is a growing body of literature on media bias as implied by the media's attempt to appeal to readers' beliefs. In addition to MS (2005), Gentzkow and Shapiro (2006) and Xiang and Sarvary (2007) also investigate this kind of bias. Gentzkow and Shapiro assume that readers who are uncertain about the quality of an information source infer that the source is of higher quality if its reports are consistent with their prior expectations. Media then slant reports toward the prior beliefs of readers to gain reputation for high quality. Xiang and Sarvary assume that there are two types of consumers, those who enjoy reading news consistent with their beliefs and conscientious consumers who care only about the truth. This assumption is different from MS (2005) or our paper, where each consumer values both some confirmation of prior beliefs and accuracy. The reporting strategy of the newspapers depends then on the relative weights consumers assign to confirmation of beliefs vs. accuracy. In addition, these earlier studies on bias assume that the media's sole source of revenue stems from selling news. In contrast, in the present study we allow the papers to earn revenues from advertising fees as well.

There are two recent papers that consider, like us, a media market with both advertising and subscription fees as sources of revenue. In Gabszewicz, Laussel and Sonnac (2002) and Ellman and Germano (2009), advertisers care only about the size and *not* the profile of the readership of each newspaper. This assumption is different from our setting, where advertisers wish to target audiences that are receptive to their advertising messages. This targeting objective of advertisers is pursued in Bergemann and Bonatti (2010) in an environment where the sole source of revenues of media outlets is from advertising. In this recent study, the authors investigate how the more accurate targeting technology that is facilitated by online versus offline advertising affects the structure of and competition among different media outlets.

Another strand of literature related to our study deals with consumers who may choose one or two of competing products. In Sarvary and Parker (1997) consumers decide whether to rely on a single information source or to diversify their purchases to include competing sources. They show that the segmentation of consumers between those who purchase one or two sources of information depends upon the relative importance consumers assign to obtaining precise information. In Guo (2006), a similar diversification of the consumption bundle may arise when there is uncertainty about future preferences. Buying competing products simultaneously "serves" as insurance against such uncertainty. As in our model, where the decision of advertisers between Double and Single-Homing depends on the horizontal location of different advertisers, in Guo as well, consumers located at the extremes of a line buy a single product and those closer to the middle buy both products. The main difference between our study and the previous two is our focus on competition between media outlets in two-sided markets instead of the one-sided framework considered in these studies.

Our work is also related to recent studies in the literature of competition among platforms in two-sided markets (e.g., Rochet and Tirole 2003, and Armstrong 2006). In our model, newspapers are platforms trying to attract both readers and advertisers. However, whereas the literature on two-sided markets focuses primarily on optimal pricing, we investigate the extent of bias in reporting that is implied by the competition for the two audiences.

Subsequent sections are organized as follows. Section 2 provides the modeling framework. In section 3, we investigate media bias when advertising fees are the sole source of revenue. In section 4 we extend the investigation to the case that newspapers derive revenues from both advertising and subscription fees. In section 5, we explore an alternative specification of the advertising response function, and in section 6 we conclude. The proofs of all Lemmas and Propositions are included in the Web Appendix.

2. THE MODEL

Consider a market with two newspapers, $i=1, 2$, a mass of A advertisers and a mass of M consumers, where M_1 of these consumers are subscribers to one of these two papers and M_2 are nonsubscribers. Newspapers provide news and print advertisements. By simultaneously operating in these two markets, newspapers have two potential sources of revenue: subscription fees (P_i) and advertising fees (K_i).

Each of the M_1 consumers reads either Newspaper 1 or 2, and may buy products from the advertisers. We adapt the model developed by MS (2005) to capture the interaction between subscribers and newspapers. Specifically, when reading the newspaper, a subscriber receives information about a certain news item t, which is distributed according to $N(0, \sigma_t^2)$. Each consumer has some prior beliefs about this news item that is distributed according to $N(b,\sigma_t^2)$. Hence, the parameter b measures the extent to which the prior beliefs of the consumer are biased relative to the true distribution of t. The bias in beliefs b (which we will simply refer to as the reader's beliefs) is uniformly distributed in the population of readers between $-b_0$ and b_0. For example, readers with beliefs closer to $-b_0$ can be considered liberals, and those in the proximity of b_0 can be considered conservatives. We assume that each of the M_1 subscribers shares information about the advertised products in the paper with relatives and friends in the M_2 population of nonsubscribers. Hence, even though only a mass of M_1 consumers are active subscribers, the entire population of consumers can be exposed to information about the products advertised in the newspapers[3].

Newspapers report news about t. They receive some data $d = t + \varepsilon$, where the random variable ε is independently distributed of t according to $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Hence, $d \sim N(0, \sigma_d^2)$, where $\sigma_d^2 = \sigma_t^2 + \sigma_\varepsilon^2$. Newspapers may choose to report the data with slant s_i, so the reported news is $n_i = d + s_i$. Readers incur disutility when reading news inconsistent with their beliefs, as measured by the distance between the reported news and the readers' beliefs: $(n_i - b)^2$. Holding constant the extent of inconsistency with their beliefs, they also prefer less slanting in the news. As in MS (2005), the overall utility of a reader is:

$$EU_b{}^i = \bar{u} - \chi E_d s_i{}^2 - \phi E_d (n_i - b)^2 - P_i \qquad \chi, \phi \geq 0, \tag{1}$$

where χ calibrates her preference for reduced slant, ϕ calibrates the reader's preference for hearing confirming news of her prior beliefs, and E_d designates the expected value operator over the random variable d.

Similar to MS (2005) we also focus on the characterization of the equilibrium with full coverage of the market[4] and linear slanting strategies for the newspapers taking the form $s_i(d) = \frac{\phi}{\phi+\chi}(B_i - d)$. The linearity of the slanting strategies is implied by our assumptions that data received is normally distributed and beliefs are uniformly distributed. The latter assumption yields linear demand functions for subscribers. It has been demonstrated in the literature that normal distribution of the random variables combined with linear demand, leads to the optimality of linear decision rules as functions of the random variables (see, for instance, Radner (1962, Theorem 5), Basar and Ho (1974), and Gal-Or, Geylani and Dukes[5] (2008)). In our setting, this finding translates to slanting strategies of the form $s_i(d) = A_0^i + A_1^i d$. Solving for the optimal coefficients, yields that $A_1^i = -\phi/(\phi + \chi)$, and as a result, A_0^i can always be expressed as $(\phi/(\phi + \chi))B_i$, with B_i interpreted as a choice of location of newspaper *i*. This location choice of the newspaper can be a point inside or outside of the interval $[-b_0, b_0]$. Notice that the extent of slanting is an increasing function of ϕ and a decreasing function of χ. Hence, as readers derive higher utility from hearing confirmatory news and reduce the importance placed on obtaining accurate information, newspapers choose greater slanting in their reporting. Without loss of generality, we assume that Newspaper 2 is located to the right of Newspaper 1 ($B_1 < B_2$). That is, while Newspaper 1 slants more to the left, Newspaper 2 slants more to the right.

For simplicity, we assume that the existence of advertising does not affect the utility derived by subscribers from reading the paper. A subscriber chooses between the two newspapers solely based upon the type of news it reports and not the type of products it advertises. Substituting the linear slanting strategies into Equation 1 and using the distributional properties of the random variable d, yields the expected payoff of a consumer having beliefs b when subscribing to newspaper i, as follows:

$$EU_b{}^i = \bar{u} - \frac{\phi^2}{\phi+\chi}(B_i - b)^2 - \frac{\chi\phi}{\phi+\chi}(b^2 + \sigma_d^2) - P_i.$$

The consumer who is indifferent between the two newspapers satisfies the equation $EU_b{}^1 = EU_b{}^2$. Solving this equation for b yields:

(2) $$b_{indif} = \frac{(\phi+\chi)}{2\phi^2}\,\frac{(P_2-P_1)}{(B_2-B_1)} + \frac{B_1+B_2}{2}.$$

Note that in the derivation of b_{indif}, we use the assumption that the accuracy of the data received by both newspapers is identical. If, in contrast, we assumed that ${\sigma_{d_1}}^2 < {\sigma_{d_2}}^2$, implying that Newspaper 1 had access to more precise information sources, the value of b_{indif} in Equation 2 would increase by $\frac{\chi}{2\phi}\,\frac{({\sigma_{d_2}}^2 - {\sigma_{d_1}}^2)}{(B_2-B_1)}$, indicating that Newspaper 1 would gain market share among subscribers[6]. Given the expression derived for $b_{indif,}$ the papers' subscription revenues are:

(3) $$R_{1,sub} = M_1 P_1 \frac{b_0 + b_{indif}}{2b_0} \text{ and } R_{2,sub} = M_1 P_2 \frac{b_0 - b_{indif}}{2b_0}.$$

The population of advertisers is distributed according to the appeal of their products to consumers having conservative beliefs, namely those situated in the positive segment of the distribution of beliefs. We designate this appeal parameter by α and assume it is uniformly distributed on the interval $[-\alpha_0, \alpha_0]$, $\alpha_0 \geq 0$. Negative values of α indicate products unappealing to conservative consumers with beliefs in the range $[0, b_0]$, with more negative values indicating increased appeal to liberal consumers with beliefs in the range $[-b_0, 0]$. Positive values of α indicate products having the opposite characteristics, with bigger positive values indicating increased appeal to conservatives. Examples of the former type of products may include hybrid cars and green products, while examples of the latter type of products may include trucks, domestic cars, and banking services for small business owners. Products whose attractiveness to the consumer is unlikely to be determined by political beliefs assume an α value in the neighborhood of zero.

We assume that advertising has the potential to enhance the expected revenue of advertised products. The possible increase in expected revenue from a given reader depends on the extent of compatibility between the beliefs of the reader (her location b) and the type of the product advertised (its appeal α). We specify the advertising response function as:

(4) $$E\,(\alpha, b) = \left(h_0 + \frac{\alpha b}{b_0}\right), \quad \text{where } h_0 > 0.$$

Hence, the effectiveness of advertising is higher when prior beliefs are more consistent with the appeal parameter of the advertised product, measured by the term αb in Equation 4. Note that the product αb is positive for both liberal consumers of products having a negative measure of appeal α and conservative consumers of products having a positive measure of appeal.

The parameter h_0 is a measure of the basic effectiveness of advertising to enhance revenues. This basic effectiveness can be modified, however, contingent upon the extent of compatibility between the variables b and α. According to Equation 4, this modification is larger for readers who have extreme political beliefs than for those who have moderate beliefs. Hence, readers who are extremely conservative respond very positively to products viewed as appealing to conservatives (e.g. light trucks) and very negatively to products viewed as appealing to liberals (e.g. green products). In contrast, moderates tend to respond only moderately (either positively or negatively) to either type of product. The specification implies that an advertiser is likely to pursue two objectives in designing its advertising strategy: to obtain a large audience for its ads and to target an audience that is receptive to its advertising message. The first component of the advertising response function in Equation 4 motivates the large audience objective and the second motivates the targeting objective. Finally, for simplicity, we assume that advertising has the same effect on a subscriber and nonsubscribers with whom she shares information about advertised products. This assumption is reasonable since subscribers tend to communicate with friends and relatives who normally hold similar political beliefs.

The payoff of an advertiser is measured by the expected increase in revenues net of the advertising fees paid to the newspapers. Hence, when an advertiser of appeal parameter α chooses to advertise only in Newspaper 1, its expected payoff is given as:

$$E_1(\alpha) = M\int_{-b_0}^{b_{indif}} \frac{1}{2b_0}\left(h_0 + \frac{\alpha b}{b_0}\right)db - K_1, \tag{5}$$

if it chooses to advertise only in Newspaper 2 its expected payoff is:

$$E_2(\alpha) = M\int_{b_{indif}}^{b_0} \frac{1}{2b_0}\left(h_0 + \frac{\alpha b}{b_0}\right)db - K_2, \tag{6}$$

and if it chooses to advertise in both papers its expected payoff is:

$$E_{12}(\alpha) = E_1(\alpha) + E_2(\alpha). \tag{7}$$

By choosing to advertise only in Newspaper 1, the advertiser recognizes that subscribers to this newspaper tend to have left leaning political beliefs, lying in the interval $[-b_0, b_{indif}]$. In contrast, by choosing to advertise only in Newspaper 2, the advertiser draws readers who have more right leaning beliefs, in the interval $[b_{indif}, b_0\ ]$. When advertising in both, the advertiser draws the entire population of readers.

An advertiser chooses to advertise in a single newspaper i if $E_i(\alpha) > E_{12}(\alpha)$ and $E_i(\alpha) > 0$. From Equations 5-7 it follows that for this advertiser $E_j(\alpha) < 0$ for $j \neq i$, namely the added benefit from advertising in the second newspaper falls short of the fee newspaper j charges. This may happen if the advertiser's product appeals mostly to readers having very extreme political beliefs. Advertising in a newspaper whose readership consists mostly of readers with opposing beliefs in the political spectrum may not be worthwhile to the advertiser in this case. In contrast, an advertiser whose product's appeal is not highly correlated with political beliefs (having an appeal parameter in the neighborhood of zero) may advertise in both newspapers since the added benefit from advertising in each paper is likely to be positive for this advertiser, implying that $E_{12}(\alpha) > E_i(\alpha), i = 1{,}2$. The above discussion indicates that the population of advertisers can be segmented into at most three intervals as described in Figure 1.

[Insert Figure 1 about here]

Advertised products with appeal parameter less than $\hat{\alpha}_1$ are advertised only in Newspaper 1 since the advertisers of these products try to target mostly liberals. In contrast, those with appeal parameter bigger than $\hat{\alpha}_2$ are advertised only in Newspaper 2, since advertisers wish to reach only conservative readers for such high values of appeal parameter. For intermediate values of $\alpha \in [\hat{\alpha}_1, \hat{\alpha}_2]$, advertisers choose to advertise in both newspapers. The number of segments in Figure 1 can be smaller than three. If $\hat{\alpha}_1 \geq \hat{\alpha}_2$, no advertiser chooses to advertise in both newspapers (referred to in the literature on two-sided markets as Single-Homing) and if $\hat{\alpha}_1 = -\hat{\alpha}_0$ and $\hat{\alpha}_2 = \alpha_0$ all advertisers choose to advertise in both newspapers, (Double-Homing by all advertisers).

The segmentation depicted in Figure 1 might change if the advertising response function of a given product is strongly influenced by whether the producer of a related product chooses to Single or Double-Home. For instance, if the advertising response-function of one beer manufacturer is adversely affected when its competitor chooses to place an ad in the same newspaper, the incidence of Double-Homing in Figure 1 might decline. Assuming that beer has a very moderate appeal parameter in the neighborhood of $\alpha = 0$, implies that competing brands would actually choose Single-Homing instead of the Double-Homing in the Figure. In contrast, if comparative advertising reduces consumers' uncertainty about the characteristics of different brands and enhances the advertising response function of each, the incidence of Double-Homing might increase, thus expanding this region in the Figure.

In what follows we will focus primarily on the derivation of *symmetric* equilibria with the market of advertisers fully covered. At such equilibria, $-\hat{\alpha}_1 = \hat{\alpha}_2 \geq 0$, and $-B_1 = B_2 \geq 0$. We will distinguish between three possible cases: equilibrium with Single-Homing, where each advertiser chooses to advertise in a single newspaper ($\hat{\alpha}_1 = \hat{\alpha}_2 = 0$ in Figure 1); equilibrium with Double-Homing by some advertisers, where there is a group of advertisers who choose to advertise in both newspapers ($-\alpha_0 < \hat{\alpha}_1 < \hat{\alpha}_2 < \alpha_0$ in Figure 1.), and Double-Homing by all[7], where all advertisers choose to Double-Home ($\hat{\alpha}_1 = -\alpha_0$, $\hat{\alpha}_2 = \alpha_0$).

From Equations 5-7 we can derive the expressions for $\hat{\alpha}_1$ and $\hat{\alpha}_2$ as functions of the locations and advertising fees chosen by the newspapers as follows:

(8) $$\hat{\alpha}_1 = \frac{2b_0}{b_0 + b_{indif}}\left\{\frac{2b_0K_2}{M(b_0 - b_{indif})} - h_0\right\},\ \hat{\alpha}_2 = \frac{2b_0}{b_0 - b_{indif}}\left\{h_0 - \frac{2b_0K_1}{M(b_0 + b_{indif})}\right\}.$$

The appeal parameter $\hat{\alpha}_1$ ($\hat{\alpha}_2$) characterizes an advertiser who is indifferent between advertising in Newspaper 1(2) and advertising in both newspapers (i.e., $E_2(\hat{\alpha}_1) = 0$ and $E_1(\hat{\alpha}_2) = 0$). The expression for $\hat{\alpha}_1$ depends only on the advertising fee charged by Newspaper 2 and *not* that charged by Newspaper 1. Similarly, the expression for $\hat{\alpha}_2$ depends only on K_1 and *not* K_2. This being the case since advertisers of both types $\hat{\alpha}_1$ and $\hat{\alpha}_2$ do not choose between the two newspapers. Instead, they choose whether to add a second outlet for advertising beyond the one they find most compatible with their advertising messages.

In the Single-Homing equilibrium the interior segment of Figure 1 disappears and the advertiser who is indifferent between in Newspaper 1 and 2 can be derived from Equations 5 and 6 by solving for α in the equation $E_1(\alpha) = E_2(\alpha)$:

(9) $$\alpha_{indif} = \frac{2b_0b_{indif}}{({b_0}^2 - {b_{indif}}^2)}h_0 - \frac{2{b_0}^2}{({b_0}^2 - {b_{indif}}^2)}\frac{(K_1 - K_2)}{M}.$$

The first term of Equation 9 compares the expected effectiveness of advertising in the two newspapers and the second term compares the advertising fees charged by them. From Equation 9 we obtain the advertising revenues that accrue to the newspapers in the equilibrium with Single-Homing as follows:

(10) $$R_{1,adv} = AK_1\frac{\alpha_0 + \alpha_{indif}}{2\alpha_0} \text{ and } R_{2,adv} = AK_2\frac{\alpha_0 - \alpha_{indif}}{2\alpha_0}.$$

In the Double-Homing equilibrium, the segment of the market covered by Newspaper 1 is $(\alpha_0 + \hat{\alpha}_2)/2\alpha_0$ and that covered by Newspaper 2 is $(\alpha_0 - \hat{\alpha}_1)/2\alpha_0$. As a result, the advertising revenues of the newspapers are:

(11) $$R_{1,adv} = AK_1\frac{\alpha_0 + \hat{\alpha}_2}{2\alpha_0} \text{ and } R_{2,adv} = AK_2\frac{\alpha_0 - \hat{\alpha}_1}{2\alpha_0}.$$

We formulate the decision process of the newspapers as a two stage game. In the first stage, each newspaper simultaneously announces a strategy $s_i(d)$ of how to report the news (its location B_i). In the second stage, the papers choose their prices P_i and K_i simultaneously. Subsequent to those two stages, advertisers choose where to advertise and readers decide to which newspaper to subscribe. Next, papers receive data d and report news $d + s_i(d)$. Finally, consumers read the news, get exposed to the advertisements, and form new impressions of the advertised products.

Using this framework but with no advertising, MS (2005) show that the equilibrium locations of the newspapers are $B_1^{MS} = -3b_0/2$ and $B_2^{MS} = 3b_0/2$. Hence, with subscription fees being the only source of revenues of newspapers, extreme bias in reporting, to the right by Newspaper 2 and to the left by Newspaper 1, are chosen at the equilibrium. Such extreme differentiation in reporting alleviates the extent of competition on subscription fees. In what follows, we first examine how these equilibrium locations change if the newspapers' sole source of revenues is advertising. Subsequently, we investigate the impact of introducing subscription fees as an additional source of revenue to supplement advertising.

It may be interesting to point out how bias in reporting as a vehicle to introduce differentiation between newspapers is different from other product features aimed at achieving differentiation. First, since the utility of readers depends upon two different attributes of news reports, accuracy and consistency with prior beliefs, the optimal slanting strategy incorporates the relative weights readers assign to these attributes. Second, since newspapers attempt to appeal to two different audiences, readers and advertisers, we will show that the positioning of each newspaper incorporates the inter-related effects of bias on both markets. This contrasts with most models of product differentiation, where features are chosen to appeal to a single consumer market.

3. ADVERTISING FEES ARE THE ONLY SOURCE OF REVENUE

In this section we assume that the newspaper's revenue stems only from advertising, i.e., $P_1=P_2=0$. Therefore, newspaper i's optimization problem in the second stage reduces to maximizing $R_{i,adv}$ in Equations 10 and 11 at the equilibrium with Single (Double)-Homing, respectively. The solution for the advertising fees as functions of the locations of the newspapers can be derived as follows:

Single-Homing

$$(12) \qquad K_1^S = M\left(\frac{\alpha_0\left(b_0^2-b_{indif}{}^2\right)}{2b_0^2}+\frac{b_{indif}h_0}{3b_0}\right),\ K_2^S = M\left(\frac{\alpha_0\left(b_0^2-b_{indif}{}^2\right)}{2b_0^2}-\frac{b_{indif}h_0}{3b_0}\right);$$

Double-Homing when $0<\hat{\alpha}_2<\alpha_0$

$$(13) \qquad K_1^D = \frac{M\left(b_0+b_{indif}\right)}{4b_0}\left(h_0+\frac{\alpha_0(b_0-b_{indif})}{2b_0}\right),\ \ K_2^D = \frac{M\left(b_0-b_{indif}\right)}{4b_0}\left(h_0+\frac{\alpha_0(b_0+b_{indif})}{2b_0}\right);$$

Double-Homing when $\hat{\alpha}_2=\alpha_0$ ($E_1(\alpha_0)=0$ determines K_1^D and $E_2(-\alpha_0)=0$ determines K_2^D)

$$(14) \qquad K_1^D = \frac{M\left(b_0+b_{indif}\right)}{2b_0}\left(h_0-\frac{\alpha_0(b_0-b_{indif})}{2b_0}\right),\ \ K_2^D = \frac{M\left(b_0-b_{indif}\right)}{2b_0}\left(h_0-\frac{\alpha_0(b_0+b_{indif})}{2b_0}\right);$$

where from Equation 2, $b_{indif}=\frac{B_1+B_2}{2}$.

Substituting the equilibrium fees back into Equations 10 and 11, yields the payoff functions of the first stage. Optimizing these functions with respect to B_i for newspaper *i*, leads to the equilibrium locations, reported in Proposition 1.

PROPOSITION 1. *If advertising is the newspapers' only source of revenue:*

(i) Single-Homing in an equilibrium when $\alpha_0 \geq h_0$

(ii) Double-Homing by a subset of advertisers is an equilibrium if $\frac{2}{3}h_0 < \alpha_0 \leq 2h_0$.

(iii) Double-Homing by all the advertisers is an equilibrium if $\alpha_0 \leq \frac{2}{3}h_0$.

(iv) Irrespective of the type of homing established at the equilibrium, each newspaper introduces no bias in reporting the news, namely $B_1^{*S} = B_2^{*S} = 0$.

In Figure 2, we depict the regions of α_0 that support the different types of Homing equilibria reported in Proposition 1.

[Insert Figure 2 about Here]

According to Figure 2, both Single-Homing and Double-Homing by a subset of advertisers can co-exist as equilibria for intermediate values of $\alpha_0 \in [h_0, 2h_0]$.Otherwise, the equilibrium is unique. For very large values of $\alpha_0 > 2h_0$, Single-Homing is the unique equilibrium , for intermediate values in the range $(2h_0/3,\ h_0)$, Double- Homing by a subset is the unique equilibrium, and for very small values of $\alpha_0 \leq 2h_0/3$, Double-Homing by all is the unique equilibrium. Essentially, Single-Homing becomes more likely as the extent of heterogeneity among advertisers is high and Double-Homing becomes more likely as this heterogeneity is low. As explained earlier, advertisers in our environment care both about the number and profile of readers who are exposed to their ads. When heterogeneity among advertisers is significant, targeting readers who are compatible with advertised products is very important to the advertisers. Single-Homing is more successful than Double-Homing in achieving such targeting.

Part (*iv*) of Proposition 1 states that when advertising is their only source of revenues, newspapers choose to eliminate bias in their reporting, thus leading to no differentiation between the newspapers. It appears that the objective of reaching a large audience is the predominant factor in determining the location of each newspaper. By choosing a location identical to that of the competitor, the newspaper guarantees the largest number of readers possible in this competitive setting, thus enhancing the willingness to pay of advertisers. This result of minimum differentiation is in sharp contrast to the result obtained in MS (2005), when the only source of revenues of newspapers stems from subscription fees. Extreme bias in reporting was derived in this previous study, as newspapers attempted to alleviate competition over subscription fees[8].

4. SUBSCRIPTION AND ADVERTISING FEES ARE BOTH SOURCES OF REVENUE

When both subscription and advertising revenues are available, the objectives of the newspapers are:

Single-Homing

$$(15)\qquad \pi_1 = A\frac{\alpha_0+\alpha_{indif}}{2\alpha_0}K_1 + M_1\frac{b_0+b_{indif}}{2b_0}P_1\ , \qquad \pi_2 = A\frac{\alpha_0-\alpha_{indif}}{2\alpha_0}K_2 + M_1\frac{b_0-b_{indif}}{2b_0}P_2\ ;$$

where b_{indif} and α_{indif} are given in Equations 2 and 9, respectively.

Double-Homing when $-\alpha_0 < \hat{\alpha}_1 < \hat{\alpha}_2 < \alpha_0$

$$(16)\qquad \pi_1 = A\frac{\alpha_0+\hat{\alpha}_2}{2\alpha_0}K_1 + M_1\frac{b_0+b_{indif}}{2b_0}P_1\ , \qquad \pi_2 = A\frac{\alpha_0-\hat{\alpha}_1}{2\alpha_0}K_2 + M_1\frac{b_0-b_{indif}}{2b_0}P_2\ ;$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are given by Equation 8.

Double-Homing when $\hat{\alpha}_1 = -\alpha_0,\ \hat{\alpha}_2 = \alpha_0$

$$(17)\qquad \pi_1 = AK_1 + M_1\frac{b_0+b_{indif}}{2b_0}P_1\ , \qquad \pi_2 = AK_2 + M_1\frac{b_0-b_{indif}}{2b_0}P_2\ ;$$

where K_1 and K_2 are given by Equation 14.

In the second stage, the newspapers set their fees to maximize the above objectives. If the newspapers locate symmetrically so that $-B_1 = B_2 = B$, the solution to this maximization can be obtained as follows:

Single-Homing

(18) $$P_S^{**} = \frac{4B\phi^2 b_0}{\phi+\chi} - \frac{A\,\mathrm{h}_0}{\frac{M_1}{\mathrm{M}}}, \qquad K_S^{**} = \frac{M\,\alpha_0}{2}.$$

Double-Homing when $-\alpha_0 < \hat{\alpha}_1 < \hat{\alpha}_2 < \alpha_0$

(19) $$P_D^{**} = \frac{4B\phi^2 b_0}{\phi+\chi} - \frac{A\mathrm{h}_0}{\frac{M_1}{\mathrm{M}}}\frac{1}{2\alpha_0}\left[h_0 + \frac{\alpha_0}{2}\right], \qquad K_D^{**} = \frac{M}{4}\left[h_0 + \frac{\alpha_0}{2}\right].$$

Double-Homing when $\hat{\alpha}_1 = -\alpha_0,\ \hat{\alpha}_2 = \alpha_0$

(20) $$P_D^{**} = \frac{4B\phi^2 b_0}{\phi+\chi} - \frac{A\mathrm{h}_0}{\frac{M_1}{\mathrm{M}}}, \qquad K_D^{**} = \frac{M}{2}\left[h_0 - \frac{\alpha_0}{2}\right].$$

Hence, for a fixed symmetric choice of locations, subscription fees are higher if subscribers have greater preference for reports that confirm their beliefs (bigger ϕ), smaller preference for accurate reporting (smaller χ), and are more heterogeneous (bigger b_0). Subscription fees are also higher when the advertising market is smaller (smaller A), the relative size of the population of subscribers is bigger (bigger M_1/M), and the effectiveness of advertising declines (smaller h_0). In general, the more important advertising revenues in comparison to subscription revenues, the lower the fees newspapers charge from subscribers at the symmetric equilibrium.

Substituting the equilibrium advertising fees derived in Equations 18, 19, and 20 back into Equation 8, implies that the ranges of the parameter α_0 that support the different types of Homing coincide with those obtained when advertising is the only source of revenue, and are still characterized in Figure 2. In Figure 3, we depict the schedules of the subscription and advertising fees that arise at the symmetric equilibrium under the various Homing strategies as a function of the basic effectiveness of advertising, h_0. In the Figure, we designate by SH, DH^- and DH the regions of Single-Homing, Double-Homing by some, and Double-Homing by all, respectively.

[Insert Figure 3 about Here]

Figure 3 nicely illustrates the negative feedback effect between the subscriber and advertiser markets. As advertising becomes more important as a source of revenue (when h_0 increases), newspapers can charge higher fees from advertisers, but this gives them also an incentive to compete more aggressively for subscribers by lowering subscription fees. Moreover, in the region where both Single and Double-Homing by some co-exist as equilibria ($\alpha_0/2 < h_0 < \alpha_0$), newspapers charge higher subscription and lower advertising fees under Double-Homing. Since advertising generates lower fees under Double-Homing, in this case, the newspapers raise their subscription fees.

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_i(B_i, B_j)$ and $K_i(B_i, B_j)$, as functions of arbitrary location choices selected in the first stage (not necessarily symmetric locations only). The second stage equilibrium strategies have to be substituted back into Equations 15-17 to obtain the first stage payoff functions of the newspapers. Assuming the existence of an interior equilibrium, next we compare the locations selected at the symmetric equilibrium (designated by B^{**}) to those derived when newspapers obtain revenues from advertisers only (denoted as $-B_1^* = B_2^* = B^A$) and from subscribers only (denoted as $-B_1^* = B_2^* = B^{MS}$). It is easy to see from Equations 18, 19 and 20 that in order to generate positive revenues from subscribers it is necessary that $B^{**} > 0$. Hence, $B^{**} > B^A = 0$, and the extent of differentiation increases when newspapers obtain revenues both from subscribers and advertisers in comparison to obtaining revenues from advertisers only. In contrast, we will demonstrate that the comparison with B^{MS} is ambiguous.

To illustrate this ambiguity, we focus only on the regimes of Single-Homing and Double-Homing by all advertisers. In Lemma 1, we first derive restrictions on the parameters of the model to guarantee that those regimes can be supported with positive streams of revenues from both subscribers and advertisers (namely that $B^{**} > 0$ and $P^{**} > 0$). For ease of presentation, we introduce a measure for the importance of advertising relative to subscription as a source of revenue for the papers, $T \stackrel{\text{def}}{=} (AM/M_1)\big((\phi+\chi)/(8\phi^2)\big)$, where (AM/M_1) represents the size of the advertising market relative to the subscription market and $(\phi+\chi)/(8\phi^2)$ is a measure of the importance consumers attach to accuracy relative to confirmation of prior beliefs. If consumers attach great importance to accurate reporting (i.e., $(\phi+\chi)/\phi^2$ is large), the papers cannot charge high subscription fees. Hence, if either one of the two components of *T* increases, the subscription market loses its importance as a source of revenues relative to the advertising market.

LEMMA 1. *To ensure positive subscription prices and strict differentiation between newspapers (i.e., $P^{**} > 0$ and $B^{**} > 0$):*

(i) At the Single-Homing equilibrium, $T < T^S_{max} \stackrel{\text{def}}{=} \frac{3{b_0}^2(9\alpha_0-4h_0)}{2h_0(9\alpha_0-2h_0)}$ and $\alpha_0 > h_0$.

(ii) At the equilibrium with Double-Homing by all advertisers ($\hat{\alpha}_1 = -\alpha_0,\ \hat{\alpha}_2 = \alpha_0$)

$T < T^D_{max} \stackrel{\text{def}}{=} \frac{{b_0}^2(3h_0-2\alpha_0)}{2h_0(2h_0-\alpha_0)}$ and $\alpha_0 < 2/3h_0$.

Restricting attention to the regions specified in Lemma 1, we derive the optimal locations chosen by the newspapers at the symmetric equilibrium in Equations 21 and 22.

<u>*Single-Homing by all advertisers*</u>

$$(21)\quad -B_1^{**} = B_2^{**} = B_S^{**} = \frac{3b_0}{4} + \frac{T\mathrm{h}_0\left(\frac{1}{2}+\frac{\mathrm{h}_0}{3\alpha_0}\right)}{b_0} + \sqrt{\left(\frac{3b_0}{4} + \frac{T\mathrm{h}_0}{b_0}\left(\frac{1}{2}+\frac{\mathrm{h}_0}{3\alpha_0}\right)\right)^2 - \frac{4T{\mathrm{h}_0}^2\left(1+\frac{2\mathrm{T}h_0}{3{\mathrm{b}_0}^2}\right)}{3\alpha_0}}\,.$$

<u>*Double-Homing by all advertisers*</u>

$$(22)\qquad -B_1^{**} = B_2^{**} = B_D^{**} = \frac{3b_0}{4} + T\frac{\alpha_0}{2b_0} + \sqrt{\left(\frac{3b_0}{4} + T\frac{\alpha_0}{2b_0}\right)^2 - 2T\alpha_0}\ .$$

We can use the above equations to evaluate the convergence properties of the equilibrium locations as the parameters α_0, h_0 and b_0 obtain their minimum and maximum values. We report these convergence properties in Observation 1.

OBSERVATION 1.

(i) When $\alpha_0 \to 0$ or $h_0 \to \infty$, only the Double-Homing equilibrium survives:

$$lim_{\alpha_0 \to 0}\ B_D^{**} = \frac{3b_0}{2}, \quad lim_{h_0 \to \infty}\ B_D^{**} < \frac{3b_0}{2}.$$

(ii) When $\alpha_0 \to \infty$ or $h_0 \to 0$, only the Single-Homing equilibrium survives:

$$lim_{\alpha_0 \to \infty}\ B_S^{**} = \frac{3b_0}{2} + T\frac{h_0}{b_0}, \quad lim_{h_0 \to 0}\ B_S^{**} = \frac{3b_0}{2}.$$

(iii) Positive revenues from subscribers are feasible only with strict heterogeneity among readers, namely $b_0 > 0$. When $b_0 \to \infty$, both Single-Homing and Double-Homing are feasible. In both cases newspapers choose maximum levels of bias, namely

$$lim_{b_0 \to \infty}\ B_S^{**} \to \infty, \quad lim_{b_0 \to \infty}\ B_D^{**} \to \infty.$$

In Proposition 2, we further characterize the properties of the equilibrium locations.

PROPOSITION 2. *When both advertising and subscription revenues are available to the newspapers:*

(i) At the Single-Homing equilibrium ($\alpha_0 > h_0$), $\frac{\partial B_S^{**}}{\partial T} > 0$, $\frac{\partial B_S^{**}}{\partial \alpha_0} > 0$, $\frac{\partial B_S^{**}}{\partial b_0} > 0$, the sign of $\frac{\partial B_S^{**}}{\partial h_0}$ is ambiguous.

(ii) At the Double-Homing equilibrium ($\alpha_0 < \frac{2}{3}h_0$), $\frac{\partial B_D^{**}}{\partial T} < 0$, $\frac{\partial B_D^{**}}{\partial \alpha_0} < 0$, $\frac{\partial B_D^{**}}{\partial b_0} > 0$, and $\frac{\partial B_D^{**}}{\partial h_0} < 0$.

In Figure 4, we depict the relationship between the equilibrium locations of the newspapers and the importance of advertising as a source of revenue to the newspapers as implied by Proposition 2.

[Insert Figure 4 about Here]

Figure 4 demonstrates that adding advertising as a source of revenue to supplement subscription fees intensifies bias in reporting when all advertisers Single-Home ($B_S^{**} > 3b_0/2$) and reduces bias when all advertisers Double-Home ($B_D^{**} < 3b_0/2$). To explain the above comparative statics results, it is important to recall that advertisers in our model may value both the size and profile of the readership exposed to their ads. Improved targeting is especially important at the equilibrium with Single-Homing, where advertisers attempt to deliver messages only to consumers whose beliefs are compatible with their ads. Greater differentiation between the newspapers can facilitate improved segmentation of readers, and as a result, better targeting of advertising messages. Notice, indeed, that at the Single-Homing equilibrium bias increases as the importance of the advertising market increases (as T goes up). In contrast, at the Double-Homing equilibrium, it is mostly the size and not the profile of the readers that matters to advertisers, given that they use both newspapers to reach consumers. As a result, as the advertising market becomes more important, each newspaper moves closer to its competitor's location in order to deliver a larger audience to advertisers. A similar explanation applies when the degree of heterogeneity among advertisers is larger (as α_0 increases). Such increased heterogeneity intensifies the importance of targeting at the Single-Homing equilibrium, thus leading to more extreme biases in reporting. In contrast, increased heterogeneity at the Double-Homing equilibrium puts downward pressure on advertising fees, since those fees have to guarantee that even advertisers having extreme appeal parameters value placing ads in both papers (advertisers of type $-\alpha_0$ or α_0). In order to remain attractive to such advertisers, the newspapers have to focus more heavily on offering a large audience, thus leading to reduced bias in reporting. Changes in the heterogeneity of readers lead to similar comparative statics in both types of Homing. Specifically, greater heterogeneity of beliefs (bigger values of b_0) leads to greater bias, as the newspapers adjust their locations to the more extreme biases of their readers.

A change in the parameter h_0 has an ambiguous effect on the location choice of the newspapers at the equilibrium with Single-Homing. On one hand, when advertising is more effective it becomes more important as a source of revenue. Hence, an increase in h_0 should have a similar effect on locations as an increase in T, another parameter that measures the importance of advertising. This first effect predicts that more effective advertising intensifies biases. On the other hand, when advertising effectiveness increases, targeting the “right” group of readers is less crucial to advertisers since ads can favorably affect perceptions irrespective of whether they are a good match with the readers’ beliefs. This reduced importance of targeting predicts less extreme biases of the newspapers. At the Double-Homing equilibrium, an increase in h_0 has an unambiguous effect of reducing bias. Since targeting is unimportant with Double-Homing, an increase in parameter h_0 plays the same role as the parameter T and reduces bias, as newspapers compete more aggressively to increase their readership.

We can use the results reported in Proposition 2 to conjecture how the equilibrium is likely to change in case of less than full coverage of consumers. At the Single-Homing equilibrium bias in reporting is significant. Hence, it is sensible that when the market is less than fully covered, it is consumers with moderate beliefs in the neighborhood of $b=0$ who choose to drop out of the market ($EU_b^i < 0$ for such consumers). As a result, the subscribers of each newspaper are fewer in number and more extreme in beliefs in comparison to a fully covered market. This new composition of subscribers would probably reduce even further the benefit from Double-Homing. Hence, less than full coverage increases in this case the likelihood that Single-Homing will prevail. In contrast, at the Double-Homing equilibrium bias is relatively small. As a result, it is now consumers with very extreme beliefs who are likely to drop out of the market. The population of subscribers becomes less heterogeneous, as a result, thus enhancing the benefit from Double-Homing. Less than full coverage reinforces one again the equilibrium advertising strategy that is obtained with full coverage of the market of consumers.

The discussion so far may provide the *wrong* impression that the type of Homing chosen by the advertisers is the main factor that determines the comparative statics results. In fact, it is the extent of heterogeneity among advertisers and the relative importance of advertising revenues to newspapers that are the main determinants. In order to illustrate this point we now focus on an environment where advertisers are exogenously required to Single-Home due to budgetary constraints. Hence, even when $\alpha_0 \leq h_0$, limited budgets prevent advertisers from placing ads in both newspapers. We use Equation 21 to illustrate in Figure 5 that the monotone relationship between the extent of bias and *T* (strictly increasing in case of Single-Homing) is no longer valid in this case.

[Insert Figure 5 about Here]

Figure 5 illustrates, that for a fixed Homing Strategy (Single) chosen by advertisers, when advertising is added as a source of revenue to supplement revenues from subscription fees, this added source intensifies bias when the relative importance of advertising to newspapers is modest (*T* small) and reduces bias in the opposite case. When *T* is relatively small, newspapers place a higher weight on alleviating competition on subscription fees than on attaining large audiences to deliver to advertisers. As a result, the targeting objective reinforces the objective of alleviating such competition, thus leading to greater bias in reporting. In contrast, when T is relatively big, revenues from subscription fees are modest and the newspapers focus more on delivering large audiences to advertisers. Hence, even when advertisers Single-Home, newspapers choose to reduce bias in reporting for sufficiently large values of T and small values of α_0. This possibility illustrates that the three effects introduced when advertising fees supplements subscription fees, may lead to opposite predictions contingent upon the relative magnitudes of the effects.

5. AN ALTERNATIVE ADVERTISING RESPONSE FUNCTION

In this section, we introduce an additional component in the advertising response function that depends on the slanting strategies of the newspapers. Specifically,

(23) $$E(\alpha, b, B_i) = h_0 + \frac{ab}{b_0} - \frac{\gamma(b-B_i)^2}{b_0}, \qquad \text{where } h_0, \gamma > 0.$$

The added component implies that the effectiveness of advertising is enhanced if the beliefs of the reader are better aligned with the location choice of the newspaper. There are anecdotes that suggest that $\gamma > 0$ for some media. For example, according to newshounds.com, democratic viewers boycott companies like UBS, AT&T/Blackberry, Mercedes Benz and American Movie Channel for choosing to advertise on FOX. Boycottwatch.org also reports conservative organizations boycotting advertisers of MSNBC, given that this news channel does not represent their political views.

In Proposition 3 we demonstrate that even when the newspapers rely only on advertising for revenues, they may choose to introduce some bias in their reporting with this modified response function.

PROPOSITION 3.

(i) When advertising is the only source of revenues of newspapers, there exist threshold values $0 < \alpha_0^{D^-} < \alpha_0^S < \alpha_0^{D^+}$ such that the equilibrium is characterized by Double-Homing of all advertisers if $\alpha_0 \leq \alpha_0^{D^-}$, Double-Homing by a subset of advertisers if $\alpha_0^{D^-} < \alpha_0 \leq \alpha_0^{D^+}$, and Single-Homing if $\alpha_0^S < \alpha_0$.

(ii)When newspapers choose to Single-Home they introduce bias in reporting if $\gamma > \frac{h_0}{b_0}$, where $B^{*S} = b_0 - \sqrt{\frac{b_0 h_0}{\gamma}}$. Otherwise, If $\gamma \leq \frac{h_0}{b_0}$, $B^{*S} = 0$.

(ii)When newspapers choose to Double-Home they introduce bias in reporting if $\gamma > \frac{h_0}{2b_0}$, where $B^{*D} = 2b_0 - \sqrt{2{b_0}^2 + \frac{b_0 h_0}{\gamma}}$. Otherwise, when $\gamma \leq \frac{h_0}{2b_0}$, $B^{*D} = 0$.

According to Proposition 3, product differentiation arises only if the parameter γ is sufficiently big. The minimum threshold on γ increases the bigger h_0 and the smaller b_0 are. When differentiation arises, it is more significant the bigger the values of γ and b_0, and the smaller the value of h_0. However, even in the limit when γ attains the biggest value feasible to support positive advertising effectiveness for all values of α and b (i.e. $\gamma < 4h_0/b_0$), the extent of differentiation is smaller than when advertisers rely only on subscribers for revenues. Specifically, when $\gamma = 4h_0/b_0$, $B^{*S} = B^{*D} = b_0/2 \; < \; 3b_0/2 = B^{MS}$.

6. CONCLUSION

In this paper we extend the work of MS (2005) by investigating media bias when advertising is added as a source of revenue to supplement subscription fees. We show that when their only source of revenue is advertising fees, newspapers choose to minimally differentiate in their reporting strategies. By eliminating any slanting in their reporting and appealing to readers with moderate beliefs, the newspapers can increase their readership, which allows them to command higher fees from advertisers. This result is in sharp contrast to the extreme bias reported in MS (2005).

When newspapers rely on revenues from advertising in addition to subscription fees, the additional advertising market introduces three different effects on the behavior of newspapers. First, the existence of advertising revenues puts downward pressure on subscription fees, as newspapers intensify competition for subscribers. As a result, newspapers may choose to increase polarization in order to alleviate this competition. Second, as newspapers attempt to increase their readership in order to attract advertisers, they may moderate slanting in order to appeal to readers having moderate beliefs. Third, since advertisers wish to target readers that are more receptive to their advertising messages, they may induce newspapers to seek greater distinctiveness and bias, and by doing so, offer a better match between advertisers and readers. We demonstrate that the extent of heterogeneity among advertisers and their relative importance as a source of revenue to newspapers determine which effects dominate.

In our model we simplified the interaction between the advertising and subscription markets by assuming that advertisers care about the overall number and profile of readers of a given newspaper, but subscribers do not care about the number or identity of the advertisers choosing to advertise with the paper. We conjecture that the polarizing effect of advertising is likely to intensify if subscribers experience greater disutility when exposed to a larger number of advertisements inconsistent with their tastes. Since both readers and subscribers value compatibility in this case, newspapers are likely to seek further differentiation in their reporting strategies, in order to support improved segmentation and matching between advertisers and readers.

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FOOTNOTES

[1]Apple has recently decided to completely stop advertising on the Fox Network since most of Apple's customers tend to be liberals, thus contradicting the perceived conservative bias of Fox.

http://www.washingtonpost.com/wp-dyn/content/article/2010/03/14/AR2010031402312.html?sid=ST2010031503503

http://www.pcworld.com/article/141473/mac_people_more_open_liberal_than_pc_users.html

[2]The correlation measure we introduce in the model assumes a value in the vicinity of zero if the perceptions of the product are unlikely to be correlated with political beliefs.

[3]The fraction of the total population who are subscribers (M_1/M) plays an important role in explaining the comparative statics results.

[4]We will later discuss the likely implication of less than full coverage on the characterization of the equilibrium.

[5]While Radner (1962) and Basar and Ho (1974) show this result for a simultaneous move, single stage game, Gal-Or, Geylani and Dukes (2008) extend the finding for a sequential move game consisting of two stages, similar to our environment.

[6]The utility specification in Equation 1 can potentially give rise both to vertical and horizontal differentiation between the newspapers. Given our objective of extending MS (2005) to allow for advertising as a source of revenues of newspapers, we focus only on horizontal differentiation, as the earlier study did.

[7]If the Double-Homing decision of advertisers leads to diminishing returns due to readers' saturation with ads, the size of the segment $[\hat{\alpha}_1, \hat{\alpha}_2]$ is likely to shrink.

[8]We will later allow the response function in Equation 4 to depend directly on the location choice of the newspaper. This leads to some differentiation between the newspapers even when their only source of revenues is advertising.

FIGURES

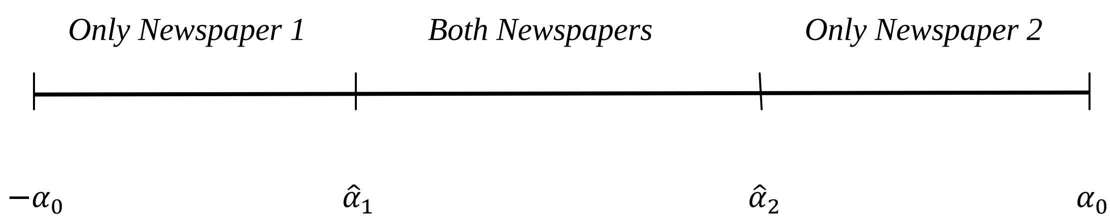

Figure 1: Segmentation of the Advertising Market

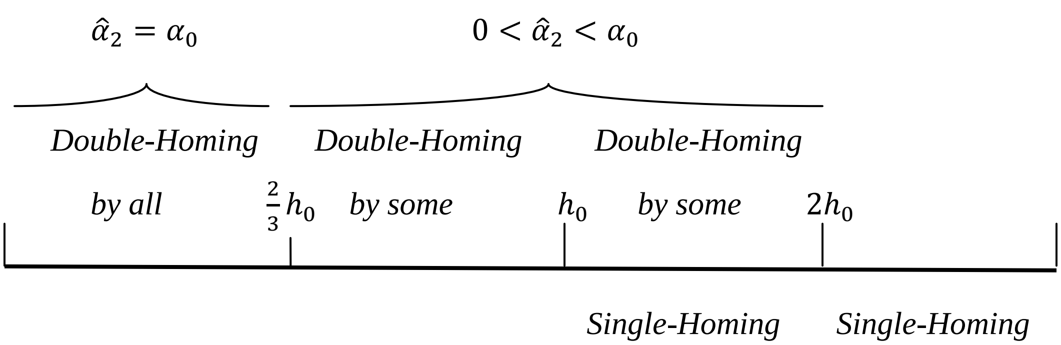

Figure 2: Regions of the Parameter α_0 that Support Single and Double-Homing

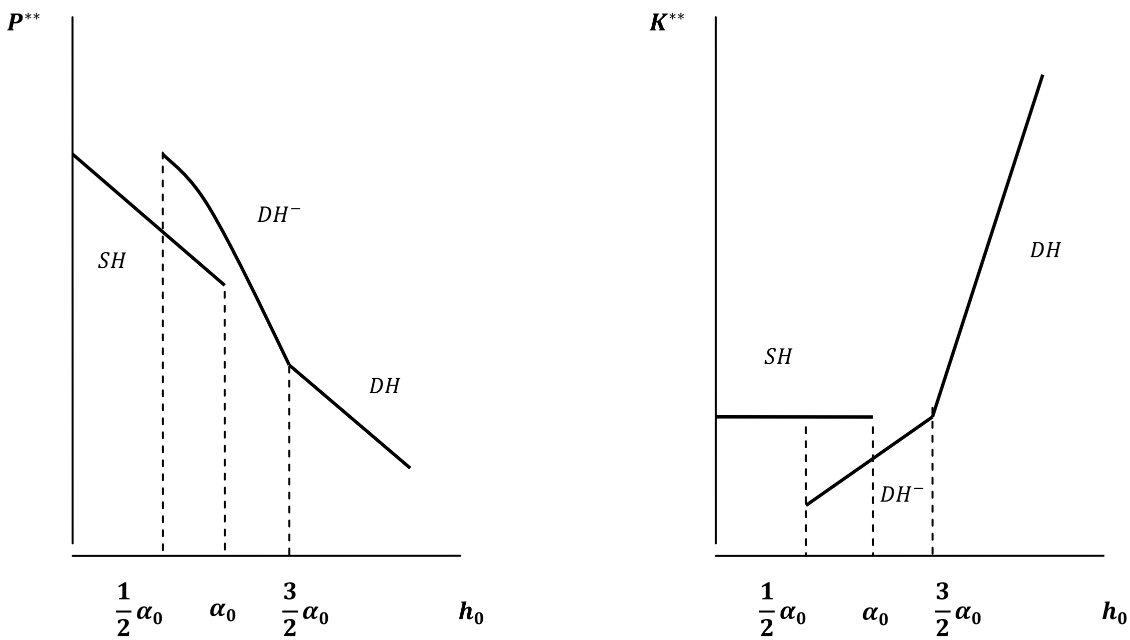

Figure 3: Equilibrium Fees as a Function of h_0

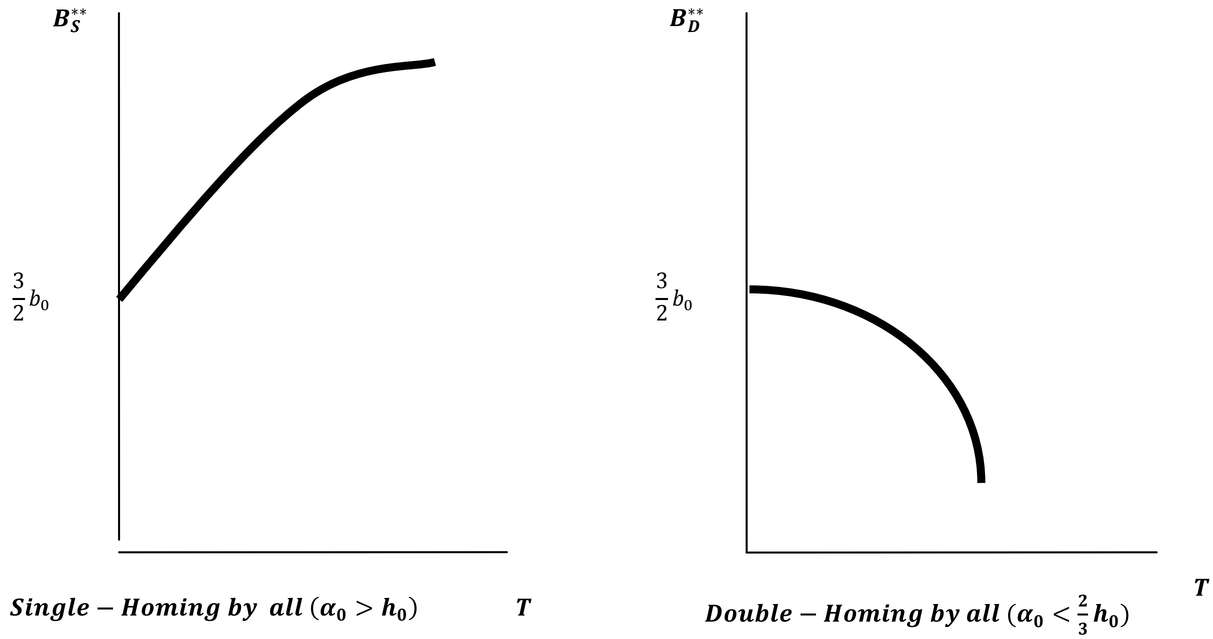

Figure 4: Equilibrium Locations as a Function of T

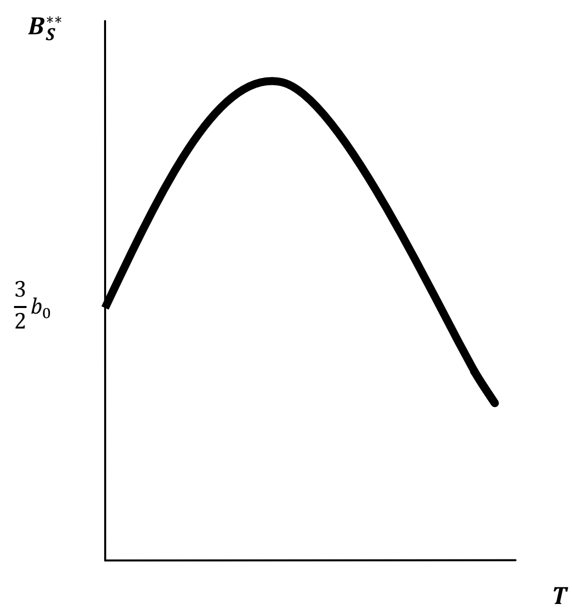

Figure 5: Locations as a Function of T when Single-Homing is Imposed

$(\boldsymbol{\alpha_0 \le h_0})$

WEB APPENDIX

Proof of Proposition 1

(i) Assuming Single-Homing equilibrium, we start with the second stage pricing decisions. Optimizing (10) with respect to the advertising fees K_1 and K_2 yields the following first order conditions:

(A1) $$\frac{\mathrm{A}}{2\alpha_0}\left(\left(\frac{\partial\alpha_{indif}}{\partial K_1}\right)K_1+(\alpha_0+\alpha_{indif})\right)=\ 0\ ,$$

(A2) $$\frac{\mathrm{A}}{2\alpha_0}\left(-\left(\frac{\partial\alpha_{indif}}{\partial K_2}\right)K_2+(\alpha_0-\alpha_{indif})\right)=\ 0.$$

It follows from (9) that,

(A3) $$\frac{\partial\alpha_{indif}}{\partial K_1}=-\frac{2\mathrm{b_0}^2}{(\mathrm{M_1}+\mathrm{M_2})(\mathrm{b_0}^2-b_{indif}{}^2)}\text{ , and}$$

(A4) $$\frac{\partial\alpha_{indif}}{\partial K_2}=\frac{2\mathrm{b_0}^2}{(\mathrm{M_1}+\mathrm{M_2})(\mathrm{b_0}^2-b_{indif}{}^2)}\ .$$

Substituting (A3), (A4) and (9) into (A1) and (A2) and solving for K_1 and K_2 , we get equilibrium advertising fees as a function of the locations as given in (12). Using (9) and the newspapers' optimal equilibrium fees (12) in (10) and optimizing with respect to B_1 and B_2 yields:

(A5) $$sgn\frac{dV_1}{dB_1}=sgn\ \left\{\frac{2b_0^2\ K_1}{\alpha_0(M_1+M_2)(b_0^2-b_{indif}{}^2)}\frac{\partial K_1}{\partial B_1}+\frac{b_0^2\ b_{indif}}{\alpha_0(M_1+M_2)\left(b_0^2-b_{indif}{}^2\right)^2}{K_1}^2\right\},$$

(A6) $$sgn\frac{dV_2}{dB_2}=sgn\ \left\{\frac{2b_0^2K_2}{\alpha_0(M_1+M_2)(b_0^2-b_{indif}{}^2)}\frac{\partial K_2}{\partial B_2}+\frac{b_0^2\ b_{indif}}{\alpha_0(M_1+M_2)\left(b_0^2-b_{indif}{}^2\right)^2}{K_2}^2\right\},$$

where *V(B₁, B₂)* is the first stage payoff function.

Evaluating the RHS of (A5) and (A6) at the symmetric equilibrium yields that $sgn\left(\frac{dV_i}{dB_i}\right)=sgn\left(\frac{\partial K_i}{\partial B_i}\right)$. Since from (12) at the symmetric equilibrium $\frac{\partial K_1}{\partial B_1}>0$ and $\frac{\partial K_2}{\partial B_2}<0$ for all values of the parameters of the model, it follows that $B_1=B_2=0$ are the equilibrium locations of the newspapers. To ensure that Single-Homing is an equilibrium, we substitute the equilibrium values of K_i at the symmetric equilibrium where $b_{indif}=0$ back into (8) to obtain $\hat{\alpha}_1=2(\alpha_0-h_0)$ and $\hat{\alpha}_2=2(h_0-\alpha_0)$. To guarantee that the interior interval in Figure 1 disappears, we impose the restriction that $\hat{\alpha}_1\geq\hat{\alpha}_2$, which happens when $\alpha_0\geq h_0$.

(ii) Assuming an equilibrium with a subset of advertisers Double-Homing, we first determine the advertising fees chosen in the second stage as a function of the location choice of the advertisers, by optimizing objectives (11). We obtain the following first order conditions:

(A7) $$(\hat{\alpha}_2 + \alpha_0) - \frac{4b_0^2 K_1}{(M_1+M_2)(b_0+b_{indif})(b_0-b_{indif})} = 0,$$

(A8) $$(\alpha_0 - \hat{\alpha}_1) - \frac{4b_0^2 K_2}{(M_1+M_2)(b_0+b_{indif})(b_0-b_{indif})} = 0.$$

Substituting for $\hat{\alpha}_1$ and $\hat{\alpha}_2$ from (8) into (A7) and (A8), yields the equilibrium fees K_1^D and K_2^D reported in (13). Substituting back into the objective function (11), we obtain that $sgn\left(\frac{dV_1}{dB_1}\right) = sgn\left(\frac{\partial\hat{\alpha}_2}{\partial B_1}\right)$ and $sgn\left(\frac{dV_2}{dB_2}\right) = -sgn\left(\frac{\partial\hat{\alpha}_1}{\partial B_2}\right)$. Since at the symmetric equilibrium $\frac{\partial\hat{\alpha}_j}{\partial B_i} > 0$ for $i, j = 1{,}2$ and $i \neq j$, it follows that $\frac{dV_1}{dB_1} > 0$ and $\frac{dV_2}{dB_2} < 0$ for all values of the parameters, thus $B_1 = B_2 = 0$, once again. To ensure that an equilibrium with a subset of advertisers exists, we need that $\alpha_0 > \hat{\alpha}_2 > \hat{\alpha}_1 > -\alpha_0$. Substituting symmetry into (13) and (8), we obtain $\hat{\alpha}_1 = \frac{\alpha_0}{2} - h_0$ and $\hat{\alpha}_2 = h_0 - \frac{\alpha_0}{2}$. Hence, $\hat{\alpha}_2 \geq \hat{\alpha}_1$ if $\alpha_0 \leq 2h_0$, and $\hat{\alpha}_2 < \alpha_0$ if $\alpha_0 > \frac{2}{3}h_0$.

(iii) At the equilibrium with Double-Homing by all advertisers, advertising fees are determined by the requirement that $E_1(\alpha_0) = 0$ and $E_2(-\alpha_0) = 0$, thus yielding (14). Substituting back into (11) when $\hat{\alpha}_1 = -\alpha_0$ and $\hat{\alpha}_2 = \alpha_0$, yields the first stage payoff functions, $V_i(B_i, B_j) = AK_i$ when all advertisers Double-Home. Hence $sgn\left(\frac{\partial V_i}{\partial B_i}\right) = sgn\left(\frac{\partial K_i}{\partial B_i}\right)$. Since at the symmetric equilibrium $\frac{\partial K_1}{\partial B_1} > 0$ and $\frac{\partial K_2}{\partial B_2} < 0$ from (14), it follows that $B_1 = B_2 = 0$. To guarantee that Double-Homing by all advertisers is an equilibrium, it follows from part (*ii*) that $\alpha_0 \leq \frac{2}{3}h_0$.

(iv) This part was established in the proofs of parts (*i*)-(*iii*).

<u>Derivations of Equations 21-22 and Proof of Lemma 1</u>

a) <u>*Single-Homing:*</u> Second stage prices are obtained by optimizing (15) with respect to P_i and K_i as follows:

(A9) $$\frac{\partial\pi_1}{\partial P_1} = \frac{A}{2\alpha_0}\left[\frac{\partial\alpha_{indif}}{\partial b_{indif}}\frac{\partial b_{indif}}{\partial P_1}K_1\right] + \frac{M_1}{2b_0}\left[\left(b_0 + b_{indif}\right) + \frac{\partial b_{indif}}{\partial P_1}P_1\right] = 0,$$

(A10) $$\frac{\partial \pi_2}{\partial P_2} = \frac{A}{2\alpha_0}\left[-\frac{\partial \alpha_{indif}}{\partial b_{indif}}\frac{\partial b_{indif}}{\partial P_2}K_2\right] + \frac{M_1}{2b_0}\left[\left(b_0 - b_{indif}\right) - \frac{\partial b_{indif}}{\partial P_2}P_2\right] = 0,$$

(A11) $$\frac{\partial \pi_1}{\partial K_1} = \frac{A}{2\alpha_0}\left[\frac{\partial \alpha_{indif}}{\partial K_1}K_1 + \left(\alpha_0 + \alpha_{indif}\right)\right] = 0 \text{ and}$$

(A12) $$\frac{\partial \pi_2}{\partial K_2} = \frac{A}{2\alpha_0}\left[-\frac{\partial \alpha_{indif}}{\partial K_2}K_2 + \left(\alpha_0 - \alpha_{indif}\right)\right] = 0.$$

From (9):

(A13) $$\frac{\partial \alpha_{indif}}{\partial b_{indif}} = \frac{2b_{indif}}{\left(\mathrm{b_0}^2 - b_{indif}{}^2\right)}\alpha_{indif} + \frac{2\mathrm{b_0 h_0}}{\left(\mathrm{b_0}^2 - b_{indif}{}^2\right)}.$$

From (2):

(A14) $$\frac{\partial b_{indif}}{\partial P_1} = \frac{\phi+\chi}{2\phi^2(\mathrm{B_1}-\mathrm{B_2})} \text{ and } \frac{\partial b_{indif}}{\partial P_2} = \frac{\phi+\chi}{2\phi^2(\mathrm{B_2}-\mathrm{B_1})}.$$

The first order conditions with respect to the advertising fees K_i coincide with those derived in the advertising case only. Since the choice of K_i does not affect the location of the indifferent consumer b_{indif} in the subscription market, optimizing π_i in (15) with respect to K_i coincides with optimizing the first term of π_i only. Hence, (12) still characterize the solution of K_i as a function of the locations B_1 and B_2. In contrast, the choice of P_i affects the subscription market directly (second term of π_i) and the advertising market indirectly (first term of π_i) since α_{indif} is a function of b_{indif}.

Substituting (2), (9), (A3), (A4), (A13) and (A14) into the first order conditions (A9)-(A12), evaluating them at symmetry ($-B_1 = B_2 = B$), and solving for K_i and P_i, we get P_S^{**} and K_S^{**} as given in (18).

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_i(B_i, B_j)$ and $K_i(B_i, B_j)$ as functions of arbitrary location choices (not only symmetric). Substituting the equilibrium strategies back into (15), we obtain the first stage payoff functions designated as $V_i(B_i, B_j)$. Differentiating with respect to the locations yields from the Envelope Theorem that:

(A15) $$\frac{\partial V_i}{\partial B_i} = \frac{\partial \pi_i}{\partial B_i} + \frac{\partial \pi_i}{\partial P_j}\frac{\partial P_j}{\partial B_i} + \frac{\partial \pi_i}{\partial K_j}\frac{\partial K_j}{\partial B_i} = 0 \qquad i,j = 1,2,\ i \neq j.$$

To illustrate the derivation of the first stage equilibrium, we focus on the optimization of Newspaper 1. For this newspaper, the terms of (A15) can be derived as follows:

(A16) $$\frac{\partial \pi_1}{\partial B_1} = \frac{M_1}{2b_0}\left(\frac{\partial b_{indif}}{\partial B_1}\right) P_1 + \frac{A}{2\alpha_0}\left(\frac{\partial \alpha_{indif}}{\partial B_1}\right)K_1,$$

(A17) $$\frac{\partial \pi_1}{\partial P_2}\frac{\partial P_2}{\partial B_1} = \frac{M_1}{2b_0}\left(\frac{\partial b_{indif}}{\partial P_2}\frac{\partial P_2}{\partial B_1}\right)P_1 + \frac{A}{2\alpha_0}\left(\frac{\partial \alpha_{indif}}{\partial b_{indif}}\frac{\partial b_{indif}}{\partial P_2}\frac{\partial P_2}{\partial B_1}\right)K_1 \text{ and}$$

(A18) $$\frac{\partial \pi_1}{\partial K_2}\frac{\partial K_2}{\partial B_1} = \frac{A}{2\alpha_0}\left(\frac{\partial \alpha_{indif}}{\partial K_2}\frac{\partial K_2}{\partial B_1}\right)K_1.$$

While the expression for $\frac{\partial K_2}{\partial B_1}$ in (A18) can be directly derived from (12), to obtain the expression from $\frac{\partial P_2}{\partial B_1}$ in (A17), we need to utilize the Implicit Function Approach by totally differentiating the first order conditions (A9) and (A10) that determine subscription fees ($\frac{\partial \pi_1}{\partial P_1} = 0$ and $\frac{\partial \pi_2}{\partial P_2} = 0$). We obtain:

(A19) $$dP_1\left(\frac{\partial^2 \pi_1}{\partial \mathrm{P_1}^2}\right) + dP_2\left(\frac{\partial^2 \pi_1}{\partial \mathrm{P_1}\partial \mathrm{P_2}}\right) + dB_1\left(\frac{\partial^2 \pi_1}{\partial \mathrm{P_1}\partial \mathrm{B_1}}\right) + dB_2\left(\frac{\partial^2 \pi_1}{\partial \mathrm{P_1}\partial \mathrm{B_2}}\right) = 0 \text{ and}$$

(A20) $$dP_1\left(\frac{\partial^2 \pi_2}{\partial \mathrm{P_1}\partial \mathrm{P_2}}\right) + dP_2\left(\frac{\partial^2 \pi_2}{\partial \mathrm{P_2}^2}\right) + dB_1\left(\frac{\partial^2 \pi_2}{\partial \mathrm{P_2}\partial \mathrm{B_1}}\right) + dB_2\left(\frac{\partial^2 \pi_2}{\partial \mathrm{P_2}\partial \mathrm{B_2}}\right) = 0.$$

From (A19) and (A20):

(A21) $$\begin{bmatrix} \frac{\partial \mathrm{P_1}}{\partial B_1} & \frac{\partial \mathrm{P_1}}{\partial B_2} \\ \frac{\partial \mathrm{P_2}}{\partial B_1} & \frac{\partial \mathrm{P_2}}{\partial B_2} \end{bmatrix} = -\begin{bmatrix} \frac{\partial^2 \pi_1}{\partial \mathrm{P_1}^2} & \frac{\partial^2 \pi_1}{\partial \mathrm{P_1}\partial \mathrm{P_2}} \\ \frac{\partial^2 \pi_2}{\partial \mathrm{P_1}\partial \mathrm{P_2}} & \frac{\partial^2 \pi_2}{\partial \mathrm{P_2}^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial^2 \pi_1}{\partial \mathrm{P_1}\partial \mathrm{B_1}} & \frac{\partial^2 \pi_1}{\partial \mathrm{P_1}\partial \mathrm{B_2}} \\ \frac{\partial^2 \pi_2}{\partial \mathrm{P_2}\partial \mathrm{B_1}} & \frac{\partial^2 \pi_2}{\partial \mathrm{P_2}\partial \mathrm{B_2}} \end{bmatrix}.$$

Using (A9) and (A10) in evaluating (A21) at the symmetric equilibrium yields:

(A22) $$\begin{bmatrix} \frac{\partial \mathrm{P_1}}{\partial B_1} & \frac{\partial \mathrm{P_1}}{\partial B_2} \\ \frac{\partial \mathrm{P_2}}{\partial B_1} & \frac{\partial \mathrm{P_2}}{\partial B_2} \end{bmatrix} = \begin{bmatrix} -\frac{\phi+\chi}{2\phi^2\mathrm{B}}(2-Z) & \frac{\phi+\chi}{2\phi^2\mathrm{B}}(1-Z) \\ \frac{\phi+\chi}{2\phi^2\mathrm{B}}(1-Z) & -\frac{\phi+\chi}{2\phi^2\mathrm{B}}(2-Z) \end{bmatrix}^{-1} \begin{bmatrix} -Y-W & -Y+W \\ Y-W & Y+W \end{bmatrix};$$

where $\mathrm{Z} \stackrel{\text{def}}{=} \frac{A(\phi+\chi)\mathrm{Mh_0}^2}{6M_1\phi^2 b_0\alpha_0 B}$, $Y \stackrel{\text{def}}{=} 1 - \frac{A\mathrm{M}(\phi+\chi)\mathrm{h_0}^2}{6M_1\alpha_0 b_0\phi^2\mathrm{B}}$ and

$W \stackrel{\text{def}}{=} -\frac{\phi+\chi}{4\phi^2\mathrm{B}^2}P^{**} - \frac{A\mathrm{M}}{M_1}\frac{(\phi+\chi)\mathrm{h_0}}{4\phi^2 B^2}$.

For second order condition, the determinant of the inverted matrix on the RHS of (A22) should be positive implying that Z< 1.5. From (A22), therefore:

(A23) $$\frac{\partial \mathrm{P_2}}{\partial B_1} = \frac{\partial \mathrm{P_1}}{\partial B_2} = \frac{2\phi^2\mathrm{B}}{\phi+\chi}\left(W - \frac{Y}{3-2Z}\right).$$

We can now complete the characterization of the optimal location choice of Newspaper 1. Using (A16) - (A18), as well as the derivation for $\frac{\partial \mathrm{K_2}}{\partial B_1}$ from (12) and $\frac{\partial \mathrm{P_2}}{\partial B_1}$ from (A23) in (A15), we obtain at the symmetric equilibrium:

(A24) $$\frac{\partial V_1}{\partial B_1} = \frac{M_1}{4b_0}P_S^{**} + \frac{M_1}{2}\frac{\partial P_2}{\partial B_1} + \frac{A}{3\alpha_0}K_S^{**}\frac{h_0}{\mathrm{b_0}} = 0, \text{ where } \frac{\partial P_2}{\partial B_1} \text{ is given by (A23).}$$

At the symmetric equilibrium when $-B_1 = B_2 = B$, we obtain from (A24) a quadratic equation as follows:

(A25) $$B^2 - B\left(\frac{3b_0}{2} + 2T\left(\frac{h_0}{3\alpha_0} + \frac{1}{2}\right)\frac{h_0}{\mathrm{b}_0}\right) + 4T\frac{\mathrm{h_0}^2}{3\alpha_0}\left(1 + \frac{2\mathrm{T}h_0}{3\mathrm{b_0}^2}\right) = 0.$$

The two roots of this quadratic equation are:

$$B_S^{**} = \frac{3b_0}{4} + T\frac{\mathrm{h}_0}{b_0}\left(\frac{1}{2} + \frac{\mathrm{h}_0}{3\alpha_0}\right) \mp \sqrt{\Delta}\text{ ; where } \Delta \stackrel{\text{def}}{=} \left(\frac{3b_0}{4} + T\frac{\mathrm{h}_0}{b_0}\left(\frac{1}{2} + \frac{\mathrm{h}_0}{3\alpha_0}\right)\right)^2 - \frac{4T\mathrm{h_0}^2}{3\alpha_0}\left(1 + \frac{2\mathrm{T}h_0}{3\mathrm{b_0}^2}\right).$$

Only the bigger root guarantees stability of reaction functions (i.e. $\frac{\partial^2 V_1}{\partial {B_1}^2} < 0$.) As a result, the optimal location at the Single-Homing equilibrium is given in (21). Note that if $\Delta < 0$ the quadratic expression (A25) is positive for all values of B. Hence, $\frac{\partial V_1}{\partial B_1} > 0$ for all B and the optimal location is the corner solution $B^{**} = 0$. Hence, $B^{**} \neq 0$ if:

(A26) $$\Delta = \left(\frac{3b_0}{4} + T\frac{\mathrm{h}_0}{b_0}\left(\frac{1}{2} + \frac{\mathrm{h}_0}{3\alpha_0}\right)\right)^2 - \frac{4T\mathrm{h_0}^2}{3\alpha_0}\left(1 + \frac{2\mathrm{T}h_0}{3\mathrm{b_0}^2}\right) > 0.$$

Inequality (A26) holds if:

(A27) $$T < \frac{3{b_0}^2\alpha_0}{2\mathrm{h}_0(2\mathrm{h}_0 - \alpha_0)}.$$

We next investigate the conditions under which P^{**} is positive. From (18):

(A28) $$P^{**} = \frac{4B\phi^2 b_0}{\phi + \chi} - \frac{A\,\mathrm{Mh}_0}{M_1} = \frac{4\phi^2}{\phi + \chi}(Bb_0 - 2T\mathrm{h}_0).$$

$P^{**} > 0$ implies $B > 2\frac{T\,\mathrm{h}_0}{b_0}$ or equivalently from (21):

(A29) $$\underbrace{\sqrt{\left(\frac{3b_0}{4} + T\left(\frac{h_0}{3\alpha_0} + \frac{1}{2}\right)\right)^2 - \frac{4T\mathrm{h_0}^2}{3\alpha_0}\left(1 + \frac{2\mathrm{T}h_0}{3\mathrm{b_0}^2}\right)}}_{LHS} > \underbrace{2\frac{T\,\mathrm{h}_0}{b_0} - \frac{3b_0}{4} - T\frac{\mathrm{h}_0}{b_0}\left(\frac{1}{2} + \frac{\mathrm{h}_0}{3\alpha_0}\right)}_{RHS}.$$

Given that the LHS is positive, there are two cases where this inequality can hold: when RHS is negative (Case 1) and when both sides are positive but the LHS is bigger (Case 2). Case 1 implies that $T < \frac{9{b_0}^2\alpha_0}{2\mathrm{h}_0(9\alpha_0 - 2\mathrm{h}_0)}$. For Case 2, squaring both sides of (A29) and solving for T yields $T < \frac{3{b_0}^2(9\alpha_0 - 4\mathrm{h}_0)}{2\mathrm{h}_0(9\alpha_0 - 2\mathrm{h}_0)}$. Combining the two cases, yields that $P^{**} > 0$ if:

(A30) $$T < \frac{3{b_0}^2(9\alpha_0 - 4\mathrm{h}_0)}{2\mathrm{h}_0(9\alpha_0 - 2\mathrm{h}_0)}.$$

Combining (A30) and (A27) yields the condition of part (*i*) of Lemma 1.

b) Double-Homing by all advertisers: Using a very similar approach to that developed when advertisers Single-Home, we obtain the following first order condition for the choice of location in the first stage.

$$\frac{d\pi_1}{dB_1} = A\left[\frac{\partial K_1}{\partial b_{indif}}\frac{\partial b_{indif}}{\partial B_1} + \frac{\partial K_1}{\partial B_1}\right] + \frac{M_1 P_1}{2b_0}\frac{\partial b_{indif}}{\partial B_1} + \left[\frac{A\partial K_1}{\partial b_{indif}}\frac{\partial b_{indif}}{\partial P_2} + \frac{M_1 P_1}{2b_0}\frac{\partial b_{indif}}{\partial P_2}\right]\frac{\partial P_2}{\partial B_1} = 0,$$

where the expression for K_1 is given in (14). At the symmetric equilibrium the above reduces to:

(A31) $$\frac{d\pi_1}{dB_1} = \frac{AM\mathrm{h}_0}{4b_0} + \frac{M_1}{2}\left[\frac{P}{2b_0} + \frac{\partial P_2}{\partial B_1}\right] = 0.$$

To derive the expression for $\frac{\partial P_2}{\partial B_1}$, we have to use, once again, the Implicit Function Approach, by totally differentiating the first order condition for the subscription fees P_i. Those conditions are:

(A32) $$\frac{\partial \pi_1}{\partial P_1} = M_1\left(b_0 + b_{indif}\right) - \frac{(\phi+\chi)}{2\phi^2(B_2-B_1)}\left\{MA\left(h_0 + \frac{\alpha_0}{h_0}b_{indif}\right) + M_1 P_1\right\} = 0,$$

$$\frac{\partial \pi_2}{\partial P_2} = M_1\left(b_0 - b_{indif}\right) - \frac{(\phi+\chi)}{2\phi^2(B_2-B_1)}\left\{\mathrm{MA}\left(\mathrm{h}_0 - \frac{\alpha_0}{h_0}b_{indif}\right) + M_1 P_2\right\} = 0.$$

Total differentiation of the first order conditions yields the following system of equations for $\frac{d\mathrm{P}_1}{dB_1}$ and $\frac{d\mathrm{P}_2}{dB_1}$: $\frac{(\phi+\chi)}{2\mathrm{B}\phi^2}\begin{bmatrix} \mathrm{R}-\mathrm{M}_1 & -\mathrm{R}+\frac{\mathrm{M}_1}{2} \\ -\mathrm{R}+\frac{\mathrm{M}_1}{2} & \mathrm{R}-\mathrm{M}_1 \end{bmatrix}\begin{bmatrix} \frac{d\mathrm{P}_1}{dB_1} \\ \frac{d\mathrm{P}_2}{dB_1} \end{bmatrix} = \begin{bmatrix} \frac{\mathrm{M}_1\mathrm{b}_0}{2\mathrm{B}} - \frac{\mathrm{M}_1}{2} + \mathrm{R} \\ \frac{\mathrm{M}_1\mathrm{b}_0}{2\mathrm{B}} + \frac{\mathrm{M}_1}{2} - \mathrm{R} \end{bmatrix}$ where $R \stackrel{\text{def}}{=} \frac{M_1 T\alpha_0}{b_0 B}$ and for second order conditions $R < \frac{3}{4}M_1$ or $T < \frac{3}{4}\frac{b_0 B}{\alpha_0}$. Solving for $\frac{d\mathrm{P}_2}{dB_1}$, we obtain: $\frac{d\mathrm{P}_2}{dB_1} = \frac{2\phi^2}{(\phi+\chi)}\left[-b_0 + \frac{\frac{B}{2}\left(\frac{T\alpha_0}{b_0} - \frac{B}{2}\right)}{\frac{3}{4}B - \frac{T\alpha_0}{b_0}}\right]$. Substituting back into (A31), yields a quadratic equation in *B* as follows:

(A33) $$B^2 - B\left(\frac{T\alpha_0}{b_0} + \frac{3}{2}b_0\right) + 2T\alpha_0 = 0.$$

There are two roots to this equation. However, only one satisfies also the condition for stability of reaction function. It is given in equation (22). The discriminant of the solution in (22) is positive if $T < \frac{{b_0}^2}{2\alpha_0}$. As well, to guarantee that $P^{**} > 0$, it follows from (20) that $T < \frac{Bb_0}{2h_0}$. Using the expression for *B* from (22) in the last inequality, yields $T < \frac{(3h_0 - 2\alpha_0){b_0}^2}{2h_0(2h_0 - \alpha_0)}$. This is a more demanding constraint than the one necessary to insure that the discriminant is positive, thus yielding part (*ii*) of Lemma 1.

Proof of Proposition 2

Follows by differentiating the expressions for $B_S{}^{**}$ and $B_D{}^{**}$ in (21) and (22) and recalling the range of the parameters that support each of type of equilibrium.

Proof of Proposition 3

Substituting the new advertising response function in the equations $E_1(\alpha) = E_2(\alpha)$ and $E_i(\alpha) = 0$, yields new expressions for $\alpha_{indif}, \hat{\alpha}_1$ and $\hat{\alpha}_2$ as follows:

(A34) $$\alpha_{indif} = \frac{2\mathrm{b}_0 b_{indif}}{(\mathrm{b}_0{}^2 - b_{indif}{}^2)}\mathrm{h}_0 - \frac{2\mathrm{b}_0{}^2}{(\mathrm{b}_0{}^2 - b_{indif}{}^2)}\frac{(\mathrm{K}_1 - \mathrm{K}_2)}{(\mathrm{M}_1 + \mathrm{M}_2)}$$
$$-\gamma\left((\mathrm{B}_1 + \mathrm{B}_2) + \frac{\mathrm{B}_1{}^2}{\mathrm{b}_0 - b_{indif}} - \frac{\mathrm{B}_2{}^2}{\mathrm{b}_0 + b_{indif}} + \frac{2b_{indif}{}^3}{3(\mathrm{b}_0{}^2 - b_{indif}{}^2)}\right).$$

(A35) $$\hat{\alpha}_1 = \frac{2b_0}{b_0 + b_{indif}}\left\{\frac{2b_0 K_2}{M(b_0 - b_{indif})} - h_0 + \frac{\gamma\left[b_0^2 + b_{indif}^2 + b_0 b_{indif} - 3B_2(b_0 + b_{indif}) + 3B_2^2\right]}{3b_0}\right\},$$

(A36) $$\hat{\alpha}_2 = \frac{2b_0}{b_0 - b_{indif}}\left\{h_0 - \frac{\gamma\left[b_0^2 + b_{indif}^2 - b_0 b_{indif} + 3B_1(b_0 - b_{indif}) + 3B_1^2\right]}{3b_0} - \frac{2b_0 K_1}{M(b_0 + b_{indif})}\right\}.$$

a) Single-Homing: Substituting (A34) into first order conditions (A1) and (A2), yields new expressions for K_1 and K_2 as functions of the locations chosen in the first stage. Optimizing the first stage payoff function with respect to B_i yields that $\frac{\partial V_i(B_1,B_2)}{\partial B_i} = 0$ when $\frac{\partial K_i}{\partial B_i} = 0$. Evaluating $\frac{\partial V_i}{\partial B_i}$ at the symmetric equilibrium ($b_{indif} = 0$ and $-B_1 = B_2 = B$) yields:

(A37) $$\frac{\partial K_i}{\partial B_i} = M\left(\frac{h_0}{6b_0} + \frac{\gamma[2Bb_0 - b_0^2 - B^2]}{6b_0{}^2}\right) = 0$$

The solution to the quadratic equation (A37) that satisfies also stability of reaction functions is given in part (*ii*) of the Proposition. The solution is strictly positive if $\gamma > \frac{h_0}{b_0}$. To ensure that Single-Homing is an equilibrium, $\hat{\alpha}_1 > \hat{\alpha}_2$. Substituting the solution for B back into (A35) and (A36) yields that this inequality holds if:

(A38) $$\alpha_0 > \sqrt{\gamma b_0 h_0} - \frac{\gamma b_0}{3} \overset{\text{def}}{=} \alpha_0{}^S.$$

b) Double-Homing by Some: Substituting (A35) and (A36) into (A7) and (A8), yields new expressions for K_1 and K_2 as functions of the locations B_1 and B_2. Optimizing the first stage payoff functions, yields that $\frac{\partial V_i}{\partial B_i} = 0$ where $\frac{\partial \hat{\alpha}_i}{\partial B_j} = 0, i \neq j, i, j = 1{,}2$. Using (A35), (A36) yields at the symmetric equilibrium:

(A39) $$\left.\frac{\partial \hat{\alpha}_i}{\partial B_j}\right|_{-B_1=B_2=B} = \frac{h_0}{b_0} - \frac{\gamma\left[-4Bb_0+2b_0^2+B^2\right]}{{b_0}^2} = 0.$$

The solution for *B* in (A39) that satisfies also stability of reaction functions is given in part (*ii*) of the Proposition. The solution is strictly positive if $\gamma > \frac{h_0}{2b_0}$. To ensure that $\alpha_0 > \hat{\alpha}_2 > \hat{\alpha}_1 > -\alpha_0$, we substitute the solution for *B* back into (A35) and (A36) to obtain the region:

$$ {\alpha_0}^{D^-} \stackrel{\text{def}}{=} \left(2\gamma\sqrt{2{b_0}^2 + \frac{b_0 h_0}{\gamma}} - \frac{26}{9}\gamma b_0\right) < \alpha_0 < \left(6\gamma\sqrt{2{b_0}^2 + \frac{b_0 h_0}{\gamma}} - \frac{26}{3}\gamma b_0\right) \stackrel{\text{def}}{=} {\alpha_0}^{D^+}. $$

c) Double-Homing by All: When $\alpha_0 < {\alpha_0}^{D^-}$, all advertisers choose to Double-Home. The expression for the equilibrium locations coincides with that derived for Double-Homing by a subset of advertisers. We find it by identifying where $\frac{\partial K_i}{\partial B_i} = 0$.

Combining the regions established in (*a*)-(*c*), we obtain that ${\alpha_0}^{D^-} < {\alpha_0}^{S} < {\alpha_0}^{D^+}$ in the region where $\frac{h_0}{b_0} < \gamma < \frac{4h_0}{b_0}$. The last inequality is necessary to guarantee a positive advertising response for all α and *b* values.